4.1 Completed Notes

4.1: Divisibility

Definition: For two whole numbers a and b, $b \neq 0$, we say <u>b divides a</u>, written as $b \mid a$, if $a \div b$ is a whole number. Other ways to say this are "a is divisible by b", "b is a divisor of a", "a is a multiple of b", and "b is a factor of a".

Divisibility Rules: Let n be a whole number.

2 | n if and only if n ends in an even number.

3 | n if and only if the sum of the digits of n is divisible by 3.
Example: Show $3 \mid 5352$.

$5+3+5+2=15.$ $3 \mid 15$, so $3 \mid 5352$

4 | n if and only if 4 divides the last 2 digits of n.
Example: Show $4 \mid 1880$.

$4 \mid 80$, so $4 \mid 1880$

5 | n if and only if n ends in either 0 or 5.

Divisibility Rules: Let n be a whole number.

6 | n if and only if 2 | n and 3 | n.

For 7, form a new number k by taking off the last digit of n and subtracting its double from the result. Then $7 \mid n$ if and only if $7 \mid k$.
Example: Show that $7 \mid 3654$.

$365-2\cdot4=357$ $\quad$ $7 \mid 21$, so $7 \mid 357$

$35-2\cdot7=21$ $\quad$ Thus, $7 \mid 3654$

8 | n if and only if 8 divides the last 3 digits of n.

9 | n if and only if the sum of the digits of n is divisible by 9.

10 | n if and only if n ends in 0.

For 11, we form a new number k by adding then subtracting the digits of n. It is important that we consider the sign of the first digit as part of this addition and subtraction. Then $11 \mid n$ if and only if $11 \mid k$.
Example: Show that $11 \mid \overset{+}{1}\overset{-}{4}\overset{+}{8}\overset{-}{5}$.

$1-4+8-5=0$

$11 \mid 0$, so $11 \mid 1485$

Example: Determine which of the numbers 2 through 11 divide 1680. Justify each of your tests.

(2) $2 \mid 0$, so $2 \mid 1680$

(3) $1+6+8+0=15.$ $3 \mid 15$, so $3 \mid 1680.$

(4) $4 \mid 80$, so $4 \mid 1680$

(5) ends in 0, so $5 \mid 1680$

(6) $2, 3 \mid 1680$, so $6 \mid 1680$

(7) $168-2\cdot0=168$

$16-2\cdot8=0$

$7 \mid 0$, so $7 \mid 1680$

(8) $8 \mid 680$, so $8 \mid 1680$

(9) $9 \nmid 15$, so $9 \nmid 1680$

does not divide

(10) ends in 0, so $10 \mid 1680$

(11) $\overset{+}{1}\overset{-}{6}\overset{+}{8}\overset{-}{0} \to 1-6+8-0=3$

$11 \nmid 3$, so $11 \nmid 1680.$

$\boxed{2, 3, 4, 5, 6, 7, 8, 10}$

Example: Determine which of the numbers 2 through 11 divide 1680. Justify each of your tests.

(8) $8 \mid 680$, so $8 \mid 1680$

(9) $9 \nmid 15$, so $9 \nmid 1680$

does not divide

(10) ends in 0, so $10 \mid 1680$

(11) $\overset{+}{1}\overset{-}{6}\overset{+}{8}\overset{-}{0} \to 1-6+8-0=3$

$11 \nmid 3$, so $11 \nmid 1680.$

$\boxed{2, 3, 4, 5, 6, 7, 8, 10}$

Example: Determine which of the numbers 2 through 11 divide 13860. Justify each of your tests.

(2) $2 \mid 0$, so $2 \mid 13860.$

(3) $1+3+8+6+0=18$ $\quad$ $3 \mid 18$, so $3 \mid 13860.$

(4) $4 \mid 60$, so $4 \mid 13860$

(5) Ends in 0, so $5 \mid 13860.$

(6) $2, 3 \mid 13860$, so $6 \mid 13860.$

(7) $1386-2\cdot0=1386$

$138-2\cdot6=126$

$12-2\cdot6=0$

$7 \mid 0$, so $7 \mid 13860.$

Example: Determine which of the numbers 2 through 11 divide 13860. Justify each of your tests.

(8) $8 \nmid 860$, so $8 \nmid 13860$

(9) $9 \mid 18$, so $9 \mid 13860$

(10) ends in 0, so $10 \mid 13860$

(11) $\overset{+}{1}\overset{-}{3}\overset{+}{8}\overset{-}{6}\overset{+}{0} \longrightarrow 1-3+8-6+0=0$

$11 \mid 0$, so $11 \mid 13860.$